

2023

Time - 3 hours

Full Marks - 80

Answer **all groups** as per instructions.

Figures in the right hand margin indicate marks.

The symbols used have their usual meaning.

GROUP - A

1. Answer all questions and fill in the blanks as required. [1 × 12

(a) Find radius of curvature of $x^2 + y^2 = 25$.

(b) Write the formula for the radius of curvature for polar curve $r = f(\theta)$.

(c) State mean value theorem.

(d) Evaluate $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x}$.

(e) The number of asymptotes of a curve of nth degree is _____.

(f) The number of loops in the curve $r = a \sin 5\theta$ is _____.

[2]

- (g) Evaluate $\lim_{x \rightarrow 0} \frac{a^x - 1}{x}$, $a > 0$.
- (h) Find the differential equation for the straight line $y = mx$.
- (i) Find an integrating factor of $\frac{dy}{dx} + y = e^{-x}$.
- (j) The n th term in the expansion of $f(a + h)$ is _____.
- (k) If $\cos x = a_0 + a_1x + a_2x^2 + \dots$, the value of a_3 is _____.
- (l) The radius of curvature of the origin, if y -axis is the tangent at the origin is given by _____.

GROUP - B

2. Answer any eight of the following.

[2 × 8

- (a) Find the radius of curvature of the curve $y = e^x$ at $(0, 1)$.
- (b) Find the equation of sphere with center at $(1, 2, 3)$ and radius is 5.
- (c) Define enveloping cone of a surface.
- (d) Find asymptote for the curve $y^2 = 4ax$.

[3]

- (e) Write the condition for the symmetric to both axes.
- (f) When $f(x)$ is continuous at $x = a$.
- (g) Find value $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)$.
- (h) Solve $\frac{d^2y}{dx^2} = 0$.
- (i) Find the maximum value of $\sin x + \cos x$.
- (j) Find the value of 'c' of Rolle's theorem for the function $f(x) = \sin x$ in $[0, \pi]$.

GROUP - C

3. Answer any eight of the following. [3 × 8

- (a) Find parallel asymptotes to $(x^2 + y^2)x - ay^2 = 0$.
- (b) Trace the curve $y^2x^2 = x^2 - a^2$.
- (c) Find area of $y^2 = 4ax$ bounded by its latus rectum.
- (d) Find the length of the entire circle $x^2 + y^2 = a^2$.
- (e) Verify Rolle's theorem for $f(x) = x^2 - 3x + 2$ on $[1, 2]$.

[4]

(f) Find $\lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + \dots + n^3}{n^4}$

(g) If $z = f\left(\frac{y}{x}\right)$, show that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0$.

(h) If $f(x, y) = \frac{xy}{x+y}$, find $f_x(2, 1)$ and $f_y(2, 1)$.

(i) Solve $p^2 - 5p + 6 = 0$, $p = \frac{dy}{dx}$.

(j) Solve $(D - 2)^3 y = 0$

GROUP - D

Answer *any four* questions.

4. Show that the curve $re^\theta = a(1 + \theta)$ has no point of inflection. [7]

5. Find the equation of sphere through the circle $x^2 + y^2 + z^2 = 9$,
 $2x + 3y + 4z = 15$ and the point $(1, 2, 3)$. [7]

6. Find maximum and minimum of the function $f(x, y) = x^3 + y^3 - 3x$
 $- 12y + 20$. [7]

7. Solve $y^{IV} - 4y''' + 8y'' - 8y' + 4y = 0$ [7]

[5]

8. Solve $xy'' - y' + (1 - x)y = e^{-x}$. [7]

9. Show that $\frac{x}{1+x} < \log(1+x) < x$ for all $x > 0$. [7]

10. Let $f(x,y) = \begin{cases} xy & , \text{ if } |x| \geq |y| \\ -xy & , \text{ if } |x| < |y|, \end{cases}$

Show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$. [7]